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14. ABSTRACT

Teh long-term goal of this project is to develop advanced tools for efficient simulations of flow-structure interactions that account for random excitation and uncertain input, with emphasis on realistic three-dimensional nonlinear representation of the structures of interest. This capability will set the foundation for the development of new effective tools for uncertainty-based multidisciplinary design and optimization procedures of naval systems, in general, and flow-structures in particular, operating under uncertainty.

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FINAL REPORT

Uncertainty-based Design Methods for Flow-Structure Interactions

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LONG-TERM GOAL

The long-term goal of this project is to develop advanced tools for efficient simulations of flow-structure interactions that account for random excitation and uncertain input, with emphasis on realistic three-dimensional *nonlinear* representation of the structures of interest. This capability will set the foundation for the development of new effective tools for *uncertainty-based* multidisciplinary design and optimization procedures of naval systems, in general, and flow-structures in particular, operating under uncertainty.

OBJECTIVES

To develop a *coupled* stochastic flow-structure formulation computing explicitly the stress field at every time step. Specifically, we coupled the deterministic high-order flow solver ***NEKTAR*** to a new high-order structure code ***StressNEKTAR*** and incorporated the uncertainty formulation. This will allow us to perform accurate fatigue (damage tolerance) analysis in order to assess the fatigue life of risers and marine structures. Both solvers are based on the same discretization, i.e., *hp-type* finite elements (spectral elements) that provide the capability of controlling the numerical error adaptively. Therefore, we will be able to target directly uncertainties associated with the flow and structural parameters as well as initial and boundary conditions, in addition to the numerical discretization errors.

APPROACH

In the previous grant we set the foundations of generalied polynomial chaos (gPC) in modeling uncertainty in simulatons of flows past bluff bodies. This work is highly cited and used now in many fields of stochastic modeling. More recently, we have extended this procedure in order to overcome some of the difficulties that polynomial chaos exhibits in dealing with discontinuities and long-time integration. Specifically, we have formulated a *multi-element* generalized polynomial chaos (ME-gPC) method that adaptively decomposes the space of random inputs when the relative error in variance becomes greater than a threshold value. In each subdomain or random element, we then employ a generalized polynomial chaos expansion. We have also developed a criterion to perform such a random decomposition adaptively, and demonstrate its effectiveness in long-time integration problems. For multi-dimensional stochastic inputs, we first test for the most *sensitive* dimension (or direction) and subsequently we perform the random decomposition along that dimension first, to gain efficiency. ME-gPC is based on Galerkin projections, which may lead to strongly coupled algebraic systesm of equations which are computationally expensive. To this end, we have also developed a collocation

version, the Multi-element probabilistic collocation methods (ME-PCM), which is very effective for strongly nonlinear problems and computationally orders of magnitude faster than Monte-Carlo.

With regards to representing stochastic inputs, the Karhunen-Loeve decomposition of a stochastic process enables the analysis of structures under random loading or material properties, using available advanced deterministic codes, such as the new code ***StressNEKTAR***.

WORK COMPLETED

The main task of this project has been the development of the structure code ***StressNEKTAR*** and its coupling to our flow solver ***NEKTAR***. In previous years, we employed the commercial code StressCheck, which is only static and its non-linear capabilities are very poor and thus quite limited to VIV analysis. The new code has elasto-dynamics capabilities and has been enhanced to perform non-linear geometrical analyses for hyperelastic materials undergoing large deformations and strains. Furthermore, it was coupled to ***NEKTAR*** based on second-order stable time integrator we studied last year. Another major development is the application of ***NEKTAR*** to designing effective controlling strategies in suppressing the vortex street formed behind bluff objects.

Specifically, the following tasks have been completed:

a) We developed and verified a new high-order dynamic parallel three-dimensional solver, ***StressNEKTAR***, for solid mechanics simulation using similar discretizations and algorithms as in ***NEKTAR***. We verified the solver by comparison with the commercial code StressCheck for linear elastic analyses, and by comparison with derived analytical solutions for elasto-dynamic response. After verification, we performed dynamic analyses with realistic loads from experimental data simulating a stationary and traveling wave on the riser (outer diameter 0.5 *m*, internal diameter of 0.4 *m*, spherical radius of 0.7 *m*, and 10 *m* long).
We then enhanced the formulations to address geometrical non-linearity (i.e. large deformations and strains) and incorporated a hyperelastic constitutive model into ***StressNEKTAR***. The implementation was verified against an analytical solution for a simplified case and solutions obtained by other codes for more complex cases.
Finally, we have strongly-coupled ***StressNEKTAR*** to ***NEKTAR***, enabling us to perform fluid structure interaction (FSI) analyses based on high order FE schemes. This is the first implementation of a high-order FSI capabilities. A systematic FSI analysis of an elastic box in flow was performed for accuracy verification.

b) We studied active and passive mechanisms of controlling the vortex street formed in flow past bluff bodies, thus leading to a complete elimination of vortex-induced vibrations. Specifically, we first studied the use of combined suction/blowing around a circular cylinder and performed an optimization 3D simulation study and corresponding stability analysis. We also studied the use of ventilation holes that connect the upstream and downstream stagnation points. In both cases, we identified the key design parameters that lead to complete VIV suppression for cylindrical structures.

c) We applied the multi-element probabilistic collocation methods (ME-PCM) to a non-linear elastic problem – the riser in a flow field - in which we investigated the riser response to uncertainties in material properties and loading. This is the first application of ME-PCM in 3-D non-linear solid mechanics, demonstrating its effectiveness. Numerical results indicate that ME-PCM is computationally orders of magnitude faster than Monte-Carlo simulations.

d) We performed high Reynolds number simulations to support the experimental work of ONR grantees Prof. D. Rockwell (Lehigh University) and Prof. M. Triantafyllou (MIT). Details of the findings are presented in the next section.

RESULTS

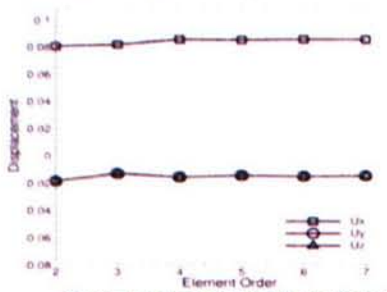
StressNEKTAR: A high-order, dynamic 3-D solid solver was developed using many of the functions and data bases of the fluid solver ***NEKTAR***. The Jacobi polynomials are used as spatial basis functions for the hexahedral elements, and mapping is performed by polynomials of an increased order. The second-order, unconditionally stable Newmark scheme, has been used for time integration. To verify the code we compared the results to these obtained by StressCheck for static loads, and the dynamic response was verified by comparison to an analytic solution. Nonlinear elasticity has been implemented in the high-order 3D structural solver, ***StressNEKTAR*** to deal with large structural deformations. The structural momentum equation (principle of virtual power) in material configuration can be expressed as:

$$\text{Seek } u \in \overset{\circ}{\mathcal{E}}{}^3(\Omega_0) \text{ such that}$$

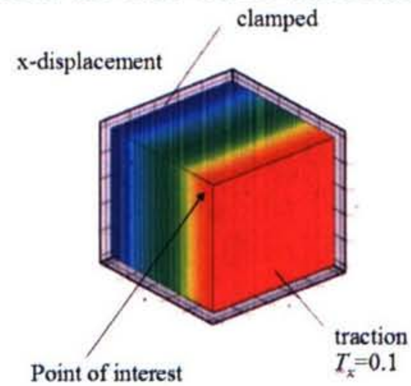
$$\int_{\Omega_0} S : \dot{E}(Q)\, d\Omega_0 - \int_{\partial\Omega_{0T}} T \cdot Q\, d\Gamma - \int_{\Omega_0} \rho_0 G \cdot Q\, d\Omega_0 + \int_{\Omega_0} \rho_0 Q \cdot \ddot{u}\, d\Omega_0 = 0 \quad \forall Q \in \overset{\circ}{\mathcal{E}}{}^3$$

where the inertial term is neglected for static nonlinearity. In the above equation, $\boldsymbol{S}$ and $\boldsymbol{E}$ are the second Piola-Kirchhoff stress tensor and the Green-Lagrange strain tensor, respectively; $\boldsymbol{T}$ is the traction vector; $\boldsymbol{G}$ is the body force;Ω_o is the undeformed 3-D domain occupied by the structure. The unknowns are the structural displacements. Since the equation is a strongly nonlinear, we employ a Newton-Raphson iterative scheme for the solution. The linear elasticity solution of the problem is used as the initial guess for the nonlinear iterations. To verify the solver we compare the results to those obtained from *AdhoC* solver. Two hexadral elements are employed to discretize the structural domain. Typically four Newton-Raphson iterations are observed for the solution to converge to

Geometric nonlinearity



Convergence w.r.t element order



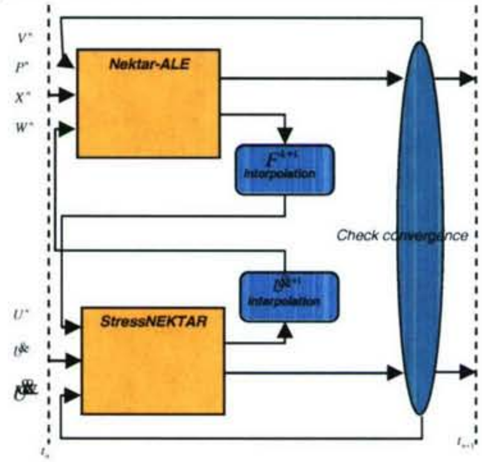
Comparison with *AdhoC* results

Element order	x-displacement AdhoC	x-displacement StressNektar	y-displacement AdhoC	y-displacement StressNektar	z-displacement AdhoC	z-displacement StressNektar
2	8.311E-2	0.08311	-1.395E-2	-0.01395	-1.395E-2	-0.01395
3	8.521E-2	0.08521	-1.425E-2	-0.01425	-1.425E-2	-0.01425

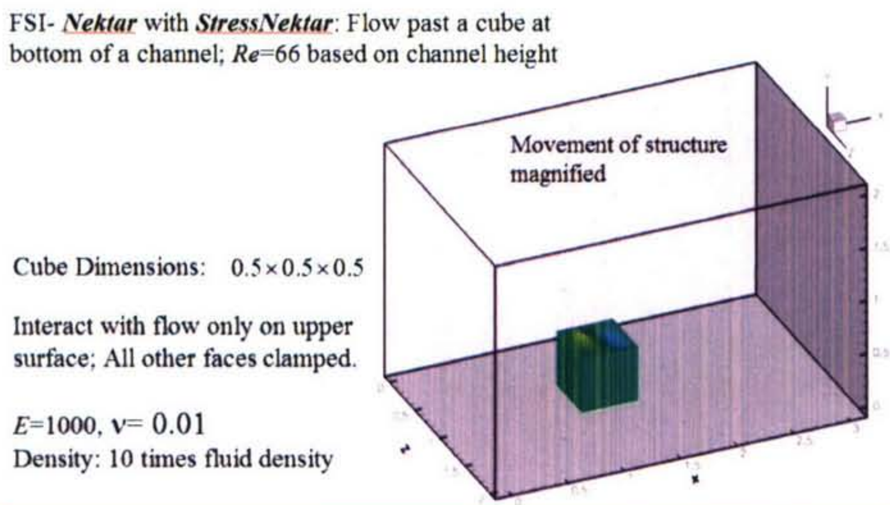
machine accuracy. The figure below shows contours of the displacement in the x direction, with blue denoting small values and red large values and we plot the displacement values at the vertex *(x,y,z)=(1.0,1.0,1.0)* as a function of the element order, demonstrating a trend of convergence of the

solutions. The displacement values at the same vertex are compared with the results from *AdhoC*; Identical results are obtained with both solvers.

Fluid Structure Interaction – NEKTAR and StressNEKTAR: The two codes have been strongly-coupled via a subiteration scheme. The pressures computed by the flow on the structure at the next time step are transferred from ***NEKTAR*** to ***StressNEKTAR***, followed by a structural analysis, which deforms the structure, and adjust the velocities of the wet interface. These now are incompatible with the "assumed" velocities and location as "seen" by ***NEKTAR***. At this instance the updated velocities of the structure's boundary are transferred from ***StressNEKTAR*** to ***NEKTAR*** and the time-step computation is redone. This subiteration process is done until convergence is achieved, as schematically shown in the right.



The FSI scheme has been used to model a flow past a cube as shown in the figure below. Although good results have been obtained, at the present time some instabilities have been detected that will have to be further investigated.



Controlling Vortex-Induced Vibrations: We have used our parallel code ***NEKTAR*** to simulate high Reynolds number flows past moving 3D rigid and flexible cylinders. In addition, we have investigated effective and practical ways of suppressing VIV of offshore structures. We present here some fundamental results that set the foundation for more practical designs for VIV control. Let us consider a 3D stationary cylinder and apply suction or blowing to it. Previous investigations have shown mixed success with this type of control but we have been able to study this problem both theoretically and computationally leading to a very effective action in completely eliminating the vortex street. Specifically, we found that by applying the proper amount of suction on the front part of the cylinder and blowing in the aft part we change the stability properties of the flow, thus producing a convectively unstable system that cannot sustain a vortex street. Typical results are shown in Figure 1 for Reynolds number 500 and 1,000. Specifically, stability analysis of the time-averaged flows was performed at different locations in the near wake where the flow is most unstable. For the no-control case we show

that the imaginary part of the eigenvalue (at the critical point) is positive, which is an indication of the well-known absolute instability. At blowing-suction of 10% (of the freestream velocity) the region of absolute instability has been reduced significantly and above 20% a convective instability emerges, which cannot sustain a vortex street. The right plot of Figure shows results from 3D DNS that confirm these results. Specifically, the RMS values of the lift coefficient are plotted – they approach zero as we reach the convective instability state, i.e. above 20% blowing-suction. We have used these results to design new passive means of controlling VIV and such results will be published in future publications.

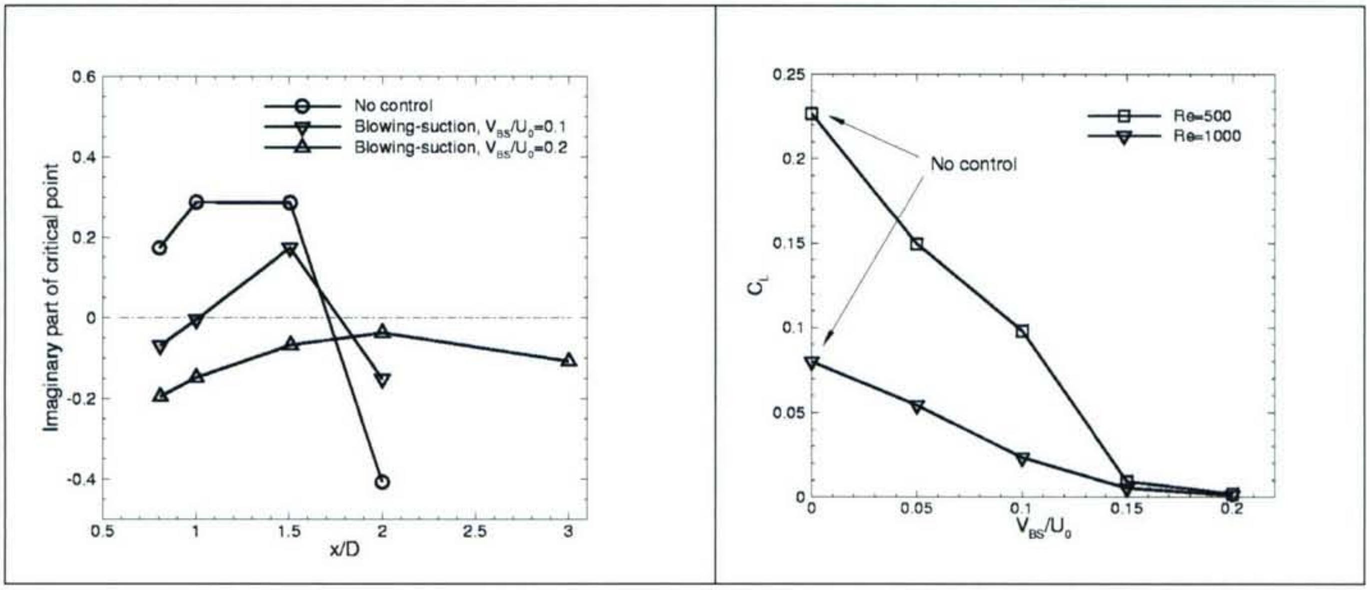


Figure: *Left – stability analysis of the cylinder average wake showing that a convective instability emerges above blowing-suction ratio of 20%. Right – Results of 3D DNS on the fluctuating lift verifying the results of stability analysis.*

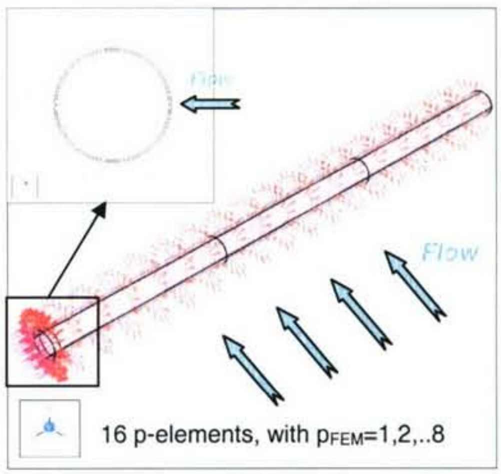
Multi-element Probabilistic Collocation Method (ME-PCM): The multi-element probabilistic collocation method is a natural extension of PCM, wherein the random space Γ is discretized into N_{PCM} nonoverlapping elements. We can then perform PCM on each element, constructing a GPC basis orthogonal on each element. If we are interested in moments and event probabilities instead of a stochastic response surface, it is not necessary to impose C^0 continuity on the boundaries of elements which have Lebesgue measure zero. The main advantage of ME-PCM is the opportunity for adaptivity. When high resolution is needed in a localized region of the domain, the grid may be refined in that region. This may be necessary, for example, when calculating failure probabilities where the general failure region is known. In addition, the number of collocation points in each element in specified region may increased so that hp-refinement is possible. In practice, this leads to computational speed-up, better accuracy for long time integration, and a more accurate treatment of solutions suffering discontinuities in the random space. For more details see the similar formulation of the Multi-Element Generalized Polynomial Chaos Method [5].

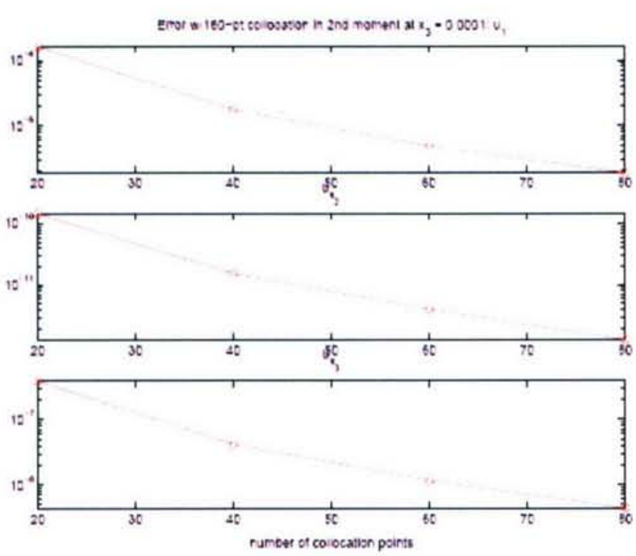
In the following examples we employ the model problem of a 3D cylindrical tube with stochastic material properties, undergoing traction forces from a realistic external flow field. The tube is clamped at one end and free at the other end. Both linear and nonlinear elasticity formulations are considered, where the nonlinearity sources are large deformations and/or strains. However, for both formulations,

the problem is nonlinear in *random* space, making it a difficult problem for stochastic galerkin spectral methods.

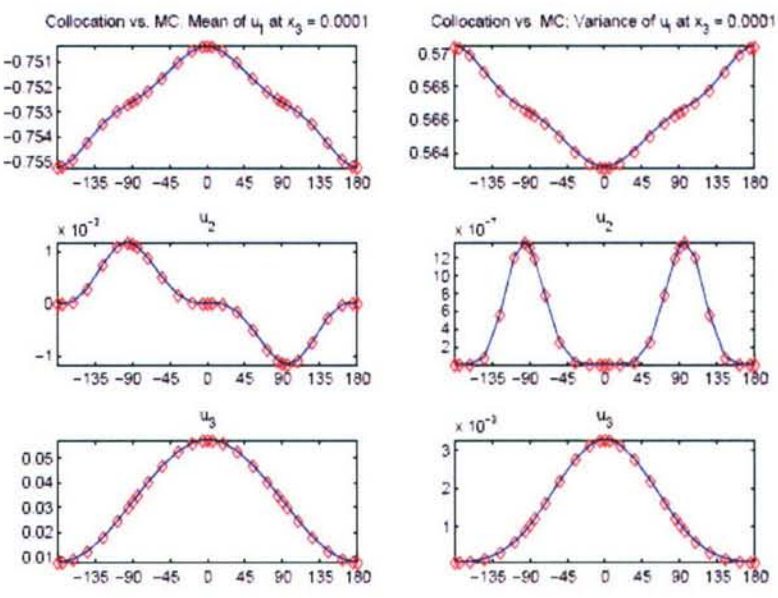
The Young's modulus $E = 212.74 + \zeta$ GPa, where ζ is uniformly distributed on [-1,1]. Here we use 80 Legendre Gauss Lobatto points for PCM, and 2000 samples for traditional Monte Carlo. The results plotted in the figures below are the mean and variance of displacement in each direction on the circumerence of the cylinder at the free end, as a function of the polar coordinate angle from the stagnation point. The PCM solution exhibits good agreement with the far more costly MC solution (blue – MC, red – PCM).

We also plot the *p*-convergence of PCM to the PCM solution with 160 points. The results shown are for the 2nd moment of displacement on the free end of the cylinder.



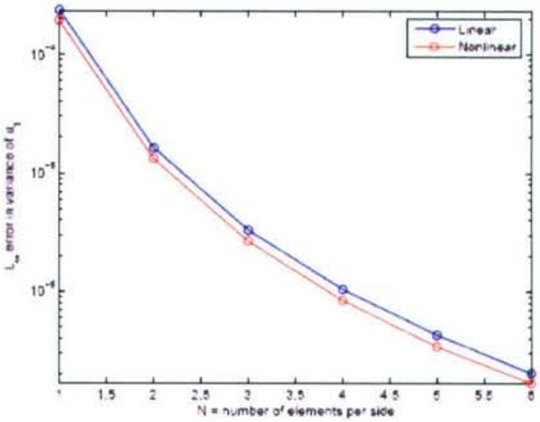


***p*-Convergence of PCM**



Monte Carlo (blue) vs PCM (red) for non-linear elasticity

ME-PCM for elasticity: In this example we choose a two dimension input: E = 212.74(1+0.15 ζ_1) + ζ_2 GPa, where ζ_1, ζ_2 are uniform random variables on [-1,1]. We use equally sized uniform grids for each decomposition of Γ = [-1,1] $\times$ [-1,1]. Since MC solution is too costly, PCM solutions are used as the 'exact' solutions. The exact solution here is PCM with 225-point grid. In the following figure, *h*-convergence of ME-PCM is shown. In these cases, 9 Legendre Gauss Lobatto points are used per element. The L_∞ error in variance of the displacement in the x-direction is plotted for both the linear and nonlinear elasticity cases. It is noted that the convergence rate does not seem to be affected by additional nonlinearities in random space.



***h*-Convergence of ME-PCM**

IMPACT/APPLICATION

Our work on flow-structure interactions is the first to address coupling of high-order 3-D non-linear solvers, and we have demonstrated that long time integration is possible and efficient with a tight control of numerical errors. The methods on gPC we have published have already been used in many applications across many fields from hydroacoustics (NRL), to modeling uncertainties in the all-electric ship (MIT/USC), to fluid mechanics and heat transfer. The new method based on probabilitic collocation is much simpler and CPU inexpensive and will also have a major impact especially for coupled problems, such as flow-structure interactions. Our work on stochastic flow past a vibrating cylinder appeared on the cover of Phys. Rev. Lett. (April 2004) and has been cited widely as a breakthrough in uncertainty quantification. The PI has edited a special volume of the Journal of Computational Physics on unceratinty quantification (September 2006).

TRANSITIONS

We have been interacting with one of the premier research groups at the Technical University of Munich and other navy labs (NRL) regarding the application of flow-structure interactions to high-order methods. We have also been interacting with several research groups regarding application of the generalized polynomial chaos approach to other fields, e.g. hydroacoustics (NRL), data assimilation (Harvard University), all-electric ship modeling (ONR consortium), solid-mechanics (the ESRD company and the CCM group of Washington University), and others. We are planning to streamline the structural code and transition it to the navy labs; interest has already been expressed by researchers at NRL for this new capability and we expect this interest to gow upon publication of our results on the nonlinear solvers and for coupled problems.

RELATED PROJECTS

During the past year, we have continued collaboration with several other ONR grantees on VIV, including M. Triantafyllou (MIT), M. Gharib (Caltech) and D. Rockwell (Lehigh University). The work with Prof. Rockwell involves detailed DNS of turbulent flows past a stationary cylinder and past an oscillating cylinder at **Re=10,000**, which is the highest Reynolds number achieved with DNS

worldwide. We have compared the DNS results of the stationary cylinder with Lehigh group's (D. Rockwell) PIV experimental data, and compared the DNS results of the oscillating cylinder with MIT group's (M. Triantafyllou) experimental data at the same Reynolds number. These comparisons show that DNS has captured the flow integral parameters (drag coefficient, lift coefficient, base pressure coefficient, Strouhal number, lift force phase angle, lift coefficient in phase with velocity) and the mean, rms and Reynolds stress statistics of the cylinder wake accurately. We have also continued our work on low-dimensional modeling of wakes in collaboration with Prof. Y. Kevrekidis of Princeton University focusing on a new promising approach, see references. This work also benefited from collaborations with Prof. Z. Yosibash (Ben-Gurion University, Israel), H. Gunes (Instabul Technical University) and D. Venturi (University of Bologna) both of whom spent several months at the PI's in the last three years.

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